LOCKING PHENOMENA IN INJECTION SYNCHRONISED PULSED OSCILLATORS

B. N. BISWAS AND G. DATTA

DEPARTMENT OF PHYSICS, UNIVERSITY OF BURDWAN, WEST BENGAL, INDIA

(*Received January* 28, 1966)

ABSTRACT. The phenomenon of synchronisation in a pulsed oscillator with a CW reference signal has been studied with particular emphasis on the locking range. The effect of low frequency time-constant of the gain control circuit of the pulsed oscillator on the locking range has been taken into consideration It has been pointed out that the broadening of the discrete spectral lines which would have otherwise occurred in an incoherent mode of oscillation will be reduced in the case studied Experimental results have been presented and found to be in good agreement with the conclusions of the analysis.

INTRODUCTION

A pulsed oscillator, as its name implies, is an oscillator the output of which consists of a series of pulsed sinusoids with a definite duty cycle A typical circuit diagram of the oscillator is shown in Fig. 1. It is to be noted that the circuit shown is essentially a super-regenerative receiver (Whitehead, J. R , 1950) and as

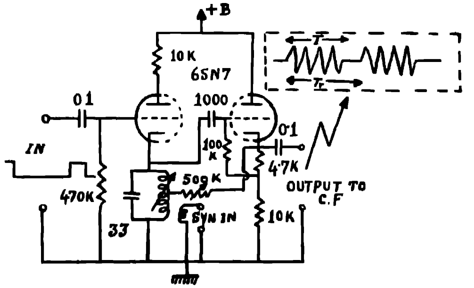

Fig. 1. A typical circuit diagram of a directly synchronised pulsed oscillator The output waveform of on-period T and total period T_r inset in the figure.

such there are two different modes of oscillations so-called logarithmic and linear depending on whether the on-period is such as to enable the oscillations to attain a steady state value in this interval or not An injection synchronised pulsed oscillator is an oscillator where an oscillation of desired frequency and amplitude is injected into the regenerative circuit. The amplitude and frequency of the injected voltage must be such as to quench the free oscillation, the quenching action being obtained through an instantaneous limiting due to which there is "strongsignal capture" and "small-signal rejection" (Chakrabarti, *et al.* 1964). It is to be noted that if there are finite transmissions through the regenerative circuit at other frequencies generated through the process of limiting then the amount of weak signal suppression obtainable is small (Biswas B N., 1964). The phenomenon of synchronization of a CW oscillator with a CW signal (Van Der pol, B, *et al.*) 1934, or with interrupted sinusoids (Fraser, D. W., 1957) is quite well-known In this paper the phenomenon of synchronisation of a pulsed oscillator with a CW signal will be studied

In section 2, the governing equations viz (i) the equation for the instantaneous amplitude of the pulsed oscillation (A) in presence of the external signal and (ii) the equation for instantaneous phase difference (Φ) between the local oscillations and the reference input have been derived A simple graphical method ($A-\Phi$ plot) has been illustrated to visualise how the range of frequency entrainment depends on the amplitude of the external signal and as well on the steady state phase difference (Φ_s) between them

Section 3 deals with derivation of an equation for the locking range of the oscillator This is followed by a study in section 4 of the effect of low frequency time-constant of the gain control circuit of the oscillator on the locking range of the pulsed oscillator

Section 5 deals with the spectral analysis of the output waveforms of (i) the coherent mode of oscillations, (ii) incoherent mode of oscillations and (iii) the synchronised mode of oscillations in a pulsed oscillator. It is suggested in this section that the broadening of the discrete spectrum which would have otherwise occurred in an incoherent mode of oscillation will be reduced in the synchronised mode of oscillations

DERIVATION OF THE GOVERNING EQUATIONS

Let us consider the equivalent analytical representation of the pulsed oscillator during the on-period as shown in Fig. 2 Let us assume that the non-linearity of the oscillator can be represented by

$$F(x) = a_1x - a_3x^3, \qquad \dots \quad (2.1)$$

where a_1 and a_3 are constants.

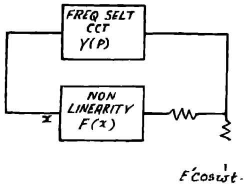

Fig. 2. Equivalent analytical representation of the directly synchronised pulsed-oscillator during the on-period with the external synchronising signal E' cos ω t.

Now for the loop shown in Fig. 2, one can write the following equations

$$\cos(\omega_1 t+\Phi)[F(A,E)+E'\cos\Phi] = \frac{A}{Y(p)} - E'\sin\Phi\sin(\omega_1 t+\Phi), \qquad (2.2)$$

where A is the instantaneous amplitude of oscillation in presence of the external signal of amplitude E' and Φ is the phase difference between them and

$$Y(p) = Y_0 \frac{\alpha p}{p^2+\alpha p+\omega_0^2} \qquad \dots \quad (2.3)$$

Now if we put $p = j\omega_1+S$ one can write $Y(P)$ as

$$\frac{1}{Y(P)} \simeq \frac{1}{Y_0}\left(1+\frac{2}{\alpha} S+j\frac{\omega_1^2-\omega_0^2}{\alpha\omega_1}\right), \qquad \dots \quad (2.4)$$

where S represents an operator in a slow time scale and $1/\alpha = Q_0/\omega_0$. Hence comparing Eq. (2.2) and (2.4) one can write

$$\frac{2}{\alpha}\frac{dA}{dt} \simeq \frac{3}{4} a_3 Y_0[A_0^2-A^2]A+E\cos\Phi, \qquad \dots \quad (2.5)$$

and

$$\frac{2}{\alpha}\frac{d\varphi}{dt} \simeq \frac{2}{\alpha}\Omega - \frac{E}{A}\sin\varphi, \qquad \dots \quad (2.6)$$

where

$$A_0^2 = \frac{a_1 Y_0 - 1}{3/4 a_3 Y_0}, \qquad \dots \quad (2.7)$$

E is the amplitude of the external signal at the input to the limiter and Ω is the instantaneous angular difference of frequency between them. From Eq. (2.7) it is quite clear that A_0 would have been the amplitude of the local oscillator if the external input were absent. In Eq. (2.5) it has tacitly been assumed that the strength of the external signal is small compared to that of the local oscillator. This equation also suggests that the oscillations in the on-period build up from and decay to noise in absence of the external signal and in presence of the external signal the oscillations build up from and decay to the reference input signal. From Eq. (2.6) it appears that during the on-period the local pulsed oscillator will be pulled towards synchronism with respect to the reference input if the following condition is satisfied :

$$K\left(=\frac{\omega_0}{2Q_0}\cdot\frac{E}{A_s}\right) \geqslant \Omega, \qquad (2.8)$$

where A_s is the steady state amplitude of the local oscillator in presence of the external signal. It is to be noted that the value of A_s depend both on E and

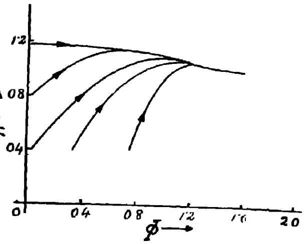

Fig. 3. Amplitude-phase plane trajectories of the directly synchronised oscillator for the case when the amplitude of the local oscillation is small compared to that of the synchronising signal.

$\Phi_s = \sin^{-1}(\Omega/K)$. An idea about the dependence of A_s on E and Φ_s can be had from the $(A-\Phi)$ plot shown in Fig 3 under the following conditions :

$$\begin{aligned} \frac{3}{4}\, Y_0(a_1) &= 1.0 \\ A_0 &= 1.0 \\ E &= 0.2 \\ \frac{2}{\alpha}\,\Omega &= 1.0 \end{aligned} \qquad (2.9)$$

Now as the maximum permissible value of ϕ_0 is 90° so it is quite clear from the $(A-\Phi)$ plot that the locking range during the on-period depends on the amplitude of free oscillation.

FORMULATION FOR THE EQUATION OF LOCKING RANGE

Before attempting to formulate an equation for the locking range it is to be remembered that the locking range is limited by the steady state amplitude of the local oscillator (vide Eq. (2.8)). This is because of the fact that during the building up of the oscillation, the external signal will have better control on the instantaneous phase of the local oscillator than when the local oscillator attains steady state. So in the synchronised condition the phase of the local oscillation will be pulled towards synchronism during the on-period and during the off-period its phase (so to say) will be deviated and the amount of this deviation will depend on the off-period. A pulsed oscillator will, therefore, be said to be in synchronism with reference to input when the local oscillator in the on-period is able to compensate for the phase deviation incurred during the off-period. That is, the net phase shift between the local oscillator and the reference over a period is zero. Expressed analytically

$$\int_0^{T_r} \left(\frac{d\varphi}{dt}\right) dt = 0 \qquad \text{.. (3.1)}$$

Now the phase equation for the on-period is given by

$$\frac{d\varphi}{dt} = \Omega - K \sin\varphi \qquad \text{... (3.2)}$$

and during the off-period it is given by

$$\frac{d\phi}{dt} = \Omega \qquad \text{... (3.3)}$$

Now at the edge of the band of synchronisation one can assume with a fair degree of accuracy that the phase difference is approximately 90°. With this condition and the condition of Eq. (3.1) in mind and solving Eq (3.2) and Eq. (3.3) one can easily write the following equation for locking range :

$$\frac{1}{\Omega/K} - \frac{\sqrt{1-(\Omega/K)^2}}{\Omega/K} \; \frac{\sqrt{\dfrac{1-\Omega/K}{1+\Omega/K}}+\tanh(nKT_r)}{1+\sqrt{\dfrac{1-\Omega/K}{1+\Omega/K}}\tanh(nKT_r)}$$

$$= \frac{1-\tan(1-n)\,\Omega T_r/2}{1+\tan(1-n)\,\Omega T_r/2}, \qquad \text{.. (3.4)}$$

where n stands for the duty cycle of the pulse sinusoids In most of the practical situations the linearised version of the Eq (3.2) which is achieved by replacing $\sin\varphi$ by $\frac{2}{\pi}\varphi$ gives a reasonable estimate of the synchron isation range that is given by

$$\frac{\Omega}{K} \simeq \frac{1-\mathrm{Exp}\left(-\frac{2}{\pi}KT\right)}{1-\mathrm{Exp}\left(-\frac{2}{\pi}KT\right)+\frac{4K}{\omega_r}(1-n)}, \qquad \text{... (3.5)}$$

where the symbols have their usual significance except $\omega_r/2\pi$ indicates repetition rate of the pulsed sinusoids.

EFFECT OF LOW FREQUENCY TIME CONSTANT OF THE GAIN CONTROL CIRCUIT OF OSCILLATOR ON THE LOCKING RANGE

So far we have tacitly assumed that the process of synchronisation is due to instantaneous limiting and thus no filtering other than at r.f. is possible (Chakrabarti 1964). But in most of the practical the gain control is only partly instantaneous. For example, the low-frequency oscillators circuits time constant in the self-bias circuit of the oscillator provides a slow acting gain control circuit. In such a case one can assume that the gain is controlled by the rectified envelope. In such a case it can be shown (op. cit.) that there will be nonlinear discrimination of one frequency against the other frequency depending upon the $R-C$ time constant of the circuit and the governing equation of the oscillator during the on-period is given by

$$\frac{d\phi}{dt} = \Omega - \frac{\omega_0}{2Q_0} \cdot \frac{E}{E_0} \alpha_0 F(p) \sin \varphi, \quad \ldots \quad (4.1)$$

where E_0 is the non-linear gain controlling voltage, α_0 is a constant that determines the rectified envelope and

$$F(p) = \frac{1}{1+p\tau} \quad \ldots \quad (4.2)$$

and τ is the R-C time constant of the grid circuit. Therefore in such a case it can be shown (op. cit.) that the maximum initial difference of frequency $(\Omega_{max}/2\pi)$ upto which the local oscillator can be made to synchronise during the on-period depends essentially on the average value of the network gain $(\overline{F(p)})$ over the range due to the initial difference of frequency $(\Omega/2\pi)$. Thus one concludes from the above discussion that variation of the locking range of a pulsed oscillator for a particular value of the repetition rate and duty cycle of the pulsed sinusoids with R-C time constant of the local oscillator will be the same as the variation of $\overline{F(p)}$ with the R-C constant.

SPECTRAL ANALYSIS OF THE OUTPUT WAVEFORM FOR DIFFERENT MODES OF OSCILLATIONS

From the discussions of the section 2, it is quite clear that the oscillations in each of the blocks of the pulsed sinusoids, (in absence of the external signal), as shown in the diagram inset in Fig. 1, build up from inherent circuit noise and as such the phases of the growing oscillations in the different blocks are completely uncorrelated with respect to the turn-on signal. This is the incoherent mode of oscillation. As a result the discrete spectrum of the output waveform is spread out into a series of broad bands (Edson, 1960). But in a synchronised mode of oscillations in a pulsed oscillator as the oscillations in each block build up from and decay to the synchronising signal so there exists a phase coherency among the oscillations in different blocks of the pulsed sinusoids in relation to the synchronising signal. It is, therefore, felt that the broadening of the discrete spectral lines which would have otherwise occurred in the incoherent mode of oscillations will be very much reduced.

Now in the coherent-mode of oscillation one can write for the output wave form

$$[e_0(t)]_{COH} = \sum_{n=-\infty}^{n=+\infty} A_c \cos \omega_c t \times U(t-nT_r), \quad n = 0, 1, 2, \text{etc.} \qquad \dots (5.1)$$

where $U(t-nT_r)$ equals step function $U(t)$, $[nT_r - \frac{1}{2}T \leqslant t \leqslant nT_r + \frac{1}{2}T]$ with unity the maximum amplitude and zero elsewhere. Similarly for the synchronised mode the output waveform can be analytically represented as

$$[e_0(t)]_{SYN} \simeq \sum_{n=-\infty}^{n=+\infty} A_c[m+U(t-nT_r)] \cos(\omega_c t+\varphi_s), \qquad \dots (5.2)$$

where φ_s is the steady state phase difference between the local oscillator and the synchronising signal and 'm' is a quantity that depends on the amount of coupling and the strength of the synchronising signal. In the incoherent mode of oscillations the output waveform can be analytically represented as

$$[e_0(t)]_{INCOH} = \sum_{n=-\infty}^{n=+\infty} A_c \cos(\omega_c t+\varphi_n) \times U(t-nT_r), \qquad \dots (5.3)$$

where φ_{+1}, φ_{+2} etc. are completely uncorrelated with respect to turn-on signal. Spectral analysis of the output waveforms [vide Eqs. (5.1), (5.2) and (5.3)] justifies the statement made elsewhere in the text.

EXPERIMENTAL RESULTS

In this section experimental results with respect to the variation of locking range of the pulsed oscillator with the frequency of interruption and the low-frequency time-constant of the gain-control circuit of the oscillator will be presented and discussed. Experimental set-up is shown in Fig. 4.

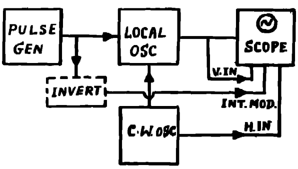

Fig. 4. Experimental set-up for measuring the variation of locking range of the pulsed oscillator with repetition rate of the pulsed oscillation of different duty cycle and low frequency time constant of the gain control circuit.

The Q-value of the tuned circuit and the coupling coil for the synchronising signal were adjusted in such a way as to have a single-peak response curve of the tuned circuit with a moderate value of Q. Presence of dip anywhere in the response curve is likely to produce spurious effect and sometimes a type of oscilla-

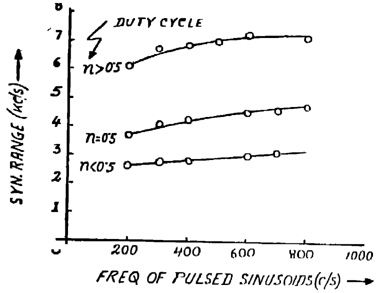

Fig 5. Pull-in performance of the directly synchronised pulsed oscillator with repetition rate and duty cycle of the pulsed oscillation

tions (Biswas, 1964). Now the amplitude of the external squagging signal is adjusted to a proper value so that the oscillator operates in the logarithmic mode (discussed elsewhere in the text). Further the amplitude of the synchronising signal is set to a minimum value so as to avoid hysteresis effect (Minorsky, 1947). Fig 5 shows the variation of the locking range with the frequency and duty cycle of the external squagging signal It will be observed that the results are in quite

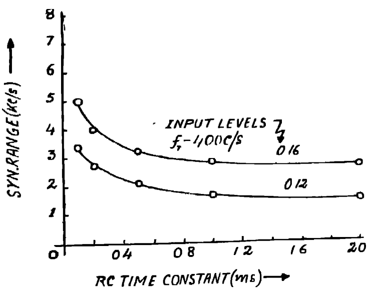

Fig 6. Locking performance of the directly synchronised pulsed oscillator with the variation of the R-C time constant of the slow acting gain-control circuit and with two different input strengths.

good agreement with the conclusions of the analysis presented in section 3. Fig 6 shows the variation of the locking range with R-C time constant of the grid circuit. These confirm the theoretical findings of section 7.

CONCLUDING REMARKS

The phenomenon of injection synchronisation of a pulsed oscillator has been analysed for continuous wave synchronising signal with a high value of CNR and its performance studied experimentally The phenomena of automatic phase control of a pulsed oscillator with respect to a CW input accompanied by an interfering signal and noises will be considered in a future communication.

ACKNOWLEDGMENT

One of the authors (B.N.B) is greatly indebted to Prof. N. B. Chakrabarti of the Indian Institute of Technology, Kharagpur, for suggesting the problem to the author. The authors take the opportunity of thanking Prof. A Mookherjee, D Sc, of Burdwan University for his kind interest and constant encouragement in this work. Thanks are also due to Mr. A K. Datta of the Burdwan University for stimulating discussions.

REFERENCES

Biswas, B. N., 1964, *Indian J. Phys.* **38**, 561.

Chakrabarti, N B and Biswas, B N, *Indian J. Phys*, 1964 **38**, 148

Edson, E. A., 1960, *Proc I.R E.*, **48**, 1454

Fraser, D. W., 1957, *Proc I.R.E.*, **45**, 1257

Minorsky, N, 1947, "*Introduction to Non-linear Mechanics*" J. W. Edwards, Ann. Arbor.

Van Der Pol, B., 1934, *Proc. I.R E.*, **22**, 1051.

Whitehead, J. R, 1950, "*Super-Regenerative Receivers*" Cambridge, At the University Press.